Modern Physics: Chapter 2 Homework

Dr. Andrew Brandt
David Joseph Stith

1. *If Planck's constant were larger than it is, would quantum phenomena be more or less conspicuous than they are now?*

   Quantum phenomena would be **more conspicuous** than they are now if Planck's constant were larger than it is. This is because quantization of energy would then occur on a larger scale and hence would appear more conspicuous.

2. (a) *A typical AM radio frequency is 1000kHz. What is the energy in eV of photons of this frequency?*

The energy of each photon is given by $E = h\nu$ where h is Planck's constant and $\nu = 1000 \times 10^3$Hz. We have

$$h \approx \frac{6.626 \times 10^{-34}\text{J} \cdot \text{s}}{1.602 \times 10^{-19}\text{J/eV}} \approx 4.136 \times 10^{-15}\text{eV} \cdot \text{s}$$

Therefore

$$E \approx (4.136 \times 10^{-15}\text{eV} \cdot \text{s})(1000 \times 10^3\text{Hz}) \approx \boxed{4.136 \times 10^{-9}\text{eV}}$$

   (b) *What is the energy of the photons in an FM signal of frequency 100 MHz?*

$$E = h\nu \approx (4.136 \times 10^{-15}\text{eV} \cdot \text{s})(100 \times 10^6\text{Hz}) \approx \boxed{4.136 \times 10^{-7}\text{eV}}$$

3. *A 1.00kW radio transmitter operates at a frequency of 880kHz. How many photons per second does it emit?*

   The energy per photon $E = h\nu \approx (6.626 \times 10^{-34}\text{J} \cdot \text{s})(880 \times 10^3\text{Hz}) \approx 5.83 \times 10^{-28}\text{J}$. Therefore the number of photons emitted per second is

$$\frac{1.00 \times 10^3\text{J/s}}{5.83 \times 10^{-28}\text{J/photon}} \approx \boxed{1.72 \times 10^{30}\text{photons/s}}$$

4. *A typical chemical bond in a biological molecule has strength of a few eV - let's say 4eV.*

   (a) *Can low intensity microwave radiation with wavelength 1cm break a bond, thus causing a mutation in a DNA molecule?*

Since $\nu = \frac{c}{\lambda}$, we have $E = h\nu = \frac{hc}{\lambda}$. Thus the energy of microwave radiation with wavelength 1cm is

$$\frac{(4.136 \times 10^{-15}\text{eV} \cdot \text{s})(3.0 \times 10^{8}\text{m/s})}{1 \times 10^{-2}\text{m}} \approx 1.24 \times 10^{-4}\text{eV}$$

This is much less than than the approximately 4eV required, therefore low intensity microwave radiation with wavelength 1cm cannot break a typical chemical bond in a biological molecule.

   (b) *What is the maximum wavelength necessary for a photon to be able to break a 4eV chemical bond?*

We have,

$$E = \frac{hc}{\lambda} \implies \lambda = \frac{hc}{E}$$

Therefore

$$\lambda \approx \frac{(4.136 \times 10^{-15}\text{eV} \cdot \text{s})(3.0 \times 10^{8}\text{m/s})}{4\text{eV}} \approx 3.1 \times 10^{-7}\text{m}$$

Therefore the maximum wavelength necessary for a photon to be able to break a 4eV chemical bond is $\boxed{3.1 \times 10^{-7}\text{m}}$.

   (c) *What type of photon has this maximum wavelength?*

The **ultraviolet** spectrum of electromagnetic radiation corresponds to this wavelength.

5. *A metal surface illuminated by 8.5×10^{14}Hz light emits electrons whose maximum energy is 0.52eV. The same surface illuminated by 12.0×10^{14}Hz light emits electrons whose maximum energy is 1.97eV. From these data find Planck's constant and the work function of the surface.*

We know that $h\nu = KE_{max} + \phi$ where ϕ and h are constant. Let $\nu_1 = 8.5 \times 10^{14}$Hz, $KE_{max_1} = 0.52$eV, and $\nu_2 = 12.0 \times 10^{14}$Hz, $KE_{max_2} = 1.97$eV. Then we have

$$\begin{aligned}
h\nu = KE_{max} + \phi &\implies \phi = h\nu - KE_{max} \\
&\implies h\nu_1 - KE_{max_1} = h\nu_2 - KE_{max_2} \\
&\implies h\nu_1 - h\nu_2 = KE_{max_1} - KE_{max_2} \\
&\implies h = \frac{KE_{max_1} - KE_{max_2}}{\nu_1 - \nu_2} \\
&\implies h \approx \frac{0.52\text{eV} - 1.97\text{eV}}{8.5 \times 10^{14}\text{Hz} - 12.0 \times 10^{14}\text{Hz}} \\
&\implies \boxed{h \approx 4.14 \times 10^{-15}\text{eV} \cdot /\text{s}}
\end{aligned}$$

Now we can use data from either frequency to obtain ϕ:

$$\begin{aligned}\phi &= h\nu_1 - KE_{max_1}\\ &\approx (4.14 \times 10^{-15}\text{eV} \cdot \text{s})(8.5 \times 10^{14}\text{Hz}) - 0.52\text{eV}\\ &\approx 3.00\text{eV}\end{aligned}$$

Therefore $\boxed{\phi \approx 3.00\text{eV}}$.

6. *Potassium chloride (KCl) has a set of crystal planes separated by distance $d = 0.31nm$. At what glancing angle to these planes would the first order Bragg maximum occur for X-rays of wavelength $0.05nm$?*

   We know that $2d\sin\theta = n\lambda$ with $d = 0.31\text{nm}$ and $n = 1$ for the first order, where θ is the glancing angle.

   Therefore $\sin\theta = \frac{\lambda}{2d}$ so that

$$\begin{aligned}\theta &= \sin^{-1}\frac{\lambda}{2d}\\ &= \sin^{-1}\left(\frac{0.05\text{nm}}{2(0.31\text{nm})}\right)\\ &\approx 4.6^\circ\end{aligned}$$

Therefore the glancing angle for the first order Bragg maximum is $\boxed{4.6^\circ}$ to the crystal planes separated by distance $d = 0.31\text{nm}$ for X-rays of wavelength 0.05nm.

7. *What is the voltage of an X-ray tube that produces X-rays with wavelengths down to $0.01nm$ but no shorter?*

   We have

$$\lambda_{min} = \frac{1.24 \times 10^{-6}\text{V} \cdot \text{m}}{V}$$

found experimentally by Duane and Hunt.

Therefore,

$$\begin{aligned}V &= \frac{1.24 \times 10^{-6}\text{V} \cdot \text{m}}{\lambda_{min}}\\ &\approx \frac{1.24 \times 10^{-6}\text{V} \cdot \text{m}}{0.01 \times 10^{-9}\text{m}}\\ &\approx 124,000\text{V}\end{aligned}$$

Therefore the voltage of an X-ray tube that produces X-rays with wavelenghts down to 0.01nm but no shorter is $\boxed{124,000\text{V}}$.

8. *An X-ray photon of initial frequency* 3.0×10^{19} *Hz collides with an electron and is scattered through* $90°$. *Find its new frequency.*

We know that $\lambda' - \lambda = \lambda_C(1 - \cos\phi)$ where λ is the initial wavelength of the photon, λ' is the new wavelength after the collision, and ϕ is the scattering angle. For an electron, λ_C is given by

$$\lambda_{C_e} = \frac{h}{m_e c} \approx \frac{6.626 \times 10^{-34}\text{J} \cdot \text{s}}{(9.109 \times 10^{-31}\text{kg})(2.998 \times 10^{8}\text{m/s})}$$
$$\approx 2.426 \times 10^{-12}\text{m}$$

Now, since $\lambda = \frac{c}{\nu}$ where ν is the frequency,

$$\lambda' - \lambda = \lambda_C(1 - \cos\phi) \Longrightarrow \lambda' = \lambda + \lambda_C(1 - \cos\phi)$$
$$\Longrightarrow \frac{c}{\nu'} = \frac{c}{\nu} + \lambda_C(1 - \cos\phi)$$
$$\Longrightarrow \nu' = \frac{c}{\frac{c}{\nu} + \lambda_C(1 - \cos\phi)}$$

Therefore, since $\cos\phi = \cos 90° = 0$,

$$\nu' \approx \frac{3.0 \times 10^{8}\text{m/s}}{\frac{3.0\times10^{8}\text{m/s}}{3.0\times10^{19}\text{Hz}} + 2.426 \times 10^{-12}\text{m}}$$
$$\approx 2.4 \times 10^{19}\text{Hz}$$

So the new frequency of an X-ray photon of initial frequency 3.0×10^{19}Hz after it collides with an electron and is scattered through $90°$ is $\boxed{2.4 \times 10^{19}\text{Hz}}$.

9. *Find the change in wavelength for photons scattered through* $180°$ *by free protons. Compare with the corresponding shift for electrons.*

We have that $\lambda' - \lambda = \lambda_C(1 - \cos\phi) = 2\lambda_C$ when $\phi = 180°$. We obtained λ_C for an electron in problem 8. For a proton,

$$\lambda_{C_p} = \frac{h}{m_p c}$$
$$\approx \frac{6.626 \times 10^{-34}\text{J} \cdot \text{s}}{(1.6726 \times 10^{-27}\text{kg})(2.998 \times 10^{8}\text{m/s})}$$
$$\approx 1.321 \times 10^{-15}\text{m}$$

Therefore the change in wavelength for photons scattered through $180°$ by free protons is $2\lambda_{C_p} = \boxed{2.642 \times 10^{-15}\text{m}}$ and by free electrons is $2\lambda_{C_e} = \boxed{4.852 \times 10^{-12}\text{m}}$.