The Famous MU Problem

MATH 3314
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Let S be the set of all strings such that $\mathbf{MI} \in S$, and for any strings x and y of zero or more letters:

$$x\mathbf{I} \in S \rightarrow x\mathbf{IU} \in S \quad (1)$$

$$\mathbf{M}x \in S \rightarrow \mathbf{M}xx \in S \quad (2)$$

$$x\mathbf{III}y \in S \rightarrow x\mathbf{U}y \in S \quad (3)$$

$$x\mathbf{UU}y \in S \rightarrow xy \in S \quad (4)$$

Furthermore, for our convenience, let $\mathbf{I}^n$ be equivalent to n consecutive occurrences of the letter I.

Example 1. $\mathbf{MUIIU} \in S$.

$$\begin{aligned}
&\mathbf{MI} \in S \\
\therefore\ &\mathbf{MII} \in S \text{ according to (2)} \\
\therefore\ &\mathbf{MI}^4 \in S \text{ according to (2)} \\
\therefore\ &\mathbf{MI}^8 \in S \text{ according to (2)} \\
\therefore\ &\mathbf{MUI}^5 \in S \text{ according to (3)} \\
\therefore\ &\mathbf{MUIIU} \in S \text{ according to (3)}
\end{aligned}$$

QED

Example 2. $\mathbf{MIIUIUUI} \in S$.

$$\begin{aligned}
&\mathbf{MI} \in S \\
\therefore\ &\mathbf{MII} \in S \text{ according to (2)} \\
\therefore\ &\mathbf{MI}^4 \in S \text{ according to (2)} \\
\therefore\ &\mathbf{MI}^8 \in S \text{ according to (2)} \\
\therefore\ &\mathbf{MI}^{16} \in S \text{ according to (2)} \\
\therefore\ &\mathbf{MI}^{16}\mathbf{U} \in S \text{ according to (1)} \\
\therefore\ &\mathbf{MI}^{13}\mathbf{UU} \in S \text{ according to (3)} \\
\therefore\ &\mathbf{MI}^{13} \in S \text{ according to (4)} \\
\therefore\ &\mathbf{MIIUI}^8 \in S \text{ according to (3)} \\
\therefore\ &\mathbf{MIIUIUIIII} \in S \text{ according to (3)} \\
\therefore\ &\mathbf{MIIUIUUI} \in S \text{ according to (3)}
\end{aligned}$$

QED

Example 3. **MIIUII** $\in S$.

$$
\begin{aligned}
&\mathbf{MI} \in S \\
\therefore\ &\mathbf{MI}^2 \in S \text{ according to (2)} \\
\therefore\ &\mathbf{MI}^4 \in S \text{ according to (2)} \\
\therefore\ &\mathbf{MI}^8 \in S \text{ according to (2)} \\
\therefore\ &\mathbf{MI}^8\mathbf{U} \in S \text{ according to (1)} \\
\therefore\ &\mathbf{MI}^5\mathbf{UU} \in S \text{ according to (3)} \\
\therefore\ &\mathbf{MI}^5 \in S \text{ according to (4)} \\
\therefore\ &\mathbf{MI}^{10} \in S \text{ according to (2)} \\
\therefore\ &\mathbf{MI}^{10}\mathbf{U} \in S \text{ according to (1)} \\
\therefore\ &\mathbf{MI}^7\mathbf{UU} \in S \text{ according to (3)} \\
\therefore\ &\mathbf{MI}^7 \in S \text{ according to (4)} \\
\therefore\ &\mathbf{MIIUII} \in S \text{ according to (3)}
\end{aligned}
$$

QED

Theorem 4. **MU** $\notin S$

Let us define the absolute value of any string x as a sum of the values of the letters **I** or **U** contained in x, where **I** is assigned the value 1 and **U** is assigned the value 3.

We claim that no element of S has a value divisible by 3, and hence **MU**, having a value of 3, is not an element of S. To show that this is true, we show that recursive application of any of the rules for producing elements of S can only yield elements whose values are congruent to 1 or $2 \pmod 3$.

The base element **MI** has a value of $1 \equiv 1 \pmod 3$. Application of rule (1), (3), or (4) to any given element results in an element whose value differs from the original by a multiple of three and hence belongs to the same congruence class modulo 3 as the original. Application of rule (2) to any element whose value is congruent to $1 \pmod 3$ results in an element whose value is congruent to $2 \pmod 3$, and applied to any element whose value is congruent to $2 \pmod 3$ results in an element whose value is congruent to $4 \equiv 1 \pmod 3$. Since there are no other production rules by which an element with a value congruent to $0 \pmod 3$ may be produced, consequently such an element cannot belong to S. Hence no element of S has a value divisible by 3.

Hence **MU** $\notin S$. **QED**