Extra Credit – Equivalent Capacitance
PHYS 1444-001

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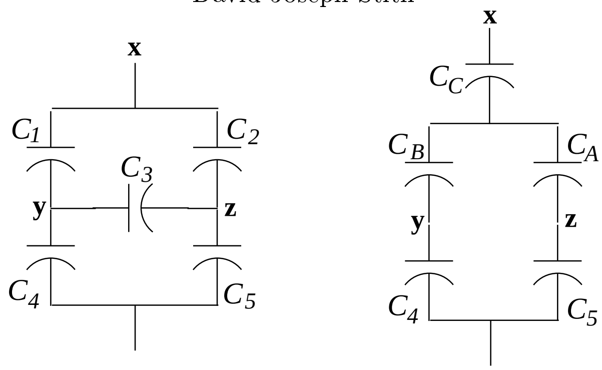

Consider the following:

$$C_A = \frac{C_1C_2 + C_2C_3 + C_1C_3}{C_1}$$
$$C_B = \frac{C_1C_2 + C_2C_3 + C_1C_3}{C_2}$$
$$C_C = \frac{C_1C_2 + C_2C_3 + C_1C_3}{C_3}$$

We first show that the C_1, C_2, C_3 'delta' complex is equivalent to the C_A, C_B, C_C 'Y' complex by showing that each of C_{xy}, C_{xz}, C_{yz} in one complex is equivalent to the corresponding pair in the other. In the 'delta' circuit, we have:

$$C_{xy} = C_1 + \frac{1}{\frac{1}{C_2} + \frac{1}{C_3}} = C_1 + \frac{C_2C_3}{C_2 + C_3} = \frac{C_1C_2 + C_1C_3 + C_2C_3}{C_2 + C_3}$$
$$C_{xz} = C_2 + \frac{1}{\frac{1}{C_1} + \frac{1}{C_3}} = C_2 + \frac{C_1C_3}{C_1 + C_3} = \frac{C_1C_2 + C_1C_3 + C_2C_3}{C_1 + C_3}$$
$$C_{yz} = C_3 + \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = C_3 + \frac{C_1C_2}{C_1 + C_2} = \frac{C_1C_2 + C_1C_3 + C_2C_3}{C_1 + C_2}$$

and in the 'Y' circuit, we have:

$$C_{xy} = \frac{1}{\frac{1}{C_B} + \frac{1}{C_C}} = \frac{1}{\frac{C_2+C_3}{C_1C_2+C_1C_3+C_2C_3}} = \frac{C_1C_2 + C_1C_3 + C_2C_3}{C_2 + C_3}$$
$$C_{xz} = \frac{1}{\frac{1}{C_A} + \frac{1}{C_C}} = \frac{1}{\frac{C_1+C_3}{C_1C_2+C_1C_3+C_2C_3}} = \frac{C_1C_2 + C_1C_3 + C_2C_3}{C_1 + C_3}$$
$$C_{yz} = \frac{1}{\frac{1}{C_A} + \frac{1}{C_B}} = \frac{1}{\frac{C_1+C_2}{C_1C_2+C_1C_3+C_2C_3}} = \frac{C_1C_2 + C_1C_3 + C_2C_3}{C_1 + C_2}$$

These are clearly equal to the 'delta', therefore the two circuits are equivalent.

Now we can find an equivalent capacitor for the entire circuit on the right using the simpler series and parallel relationships. This will then also represent an equivalent capacitor for the circuit on the left since the two circuits are equivalent.

Let $Z = C_1C_2 + C_1C_3 + C_2C_3$. Then:

$$\begin{aligned}
C_{eq} &= \frac{1}{\frac{1}{C_C} + \frac{1}{\frac{1}{\frac{1}{C_B}+\frac{1}{C_4}} + \frac{1}{\frac{1}{C_A}+\frac{1}{C_5}}}} \\
&= \frac{1}{\frac{C_3}{Z} + \frac{1}{\frac{1}{\frac{C_2}{Z}+\frac{1}{C_4}} + \frac{1}{\frac{C_1}{Z}+\frac{1}{C_5}}}} \\
&= \frac{1}{\frac{C_3}{Z} + \frac{1}{\frac{C_4Z}{Z+C_2C_4} + \frac{C_5Z}{Z+C_1C_5}}} \\
&= \frac{1}{\frac{C_3}{Z} + \frac{1}{\frac{C_4Z(Z+C_1C_5)+C_5Z(Z+C_2C_4)}{(Z+C_2C_4)(Z+C_1C_5)}}} \\
&= \frac{1}{\frac{C_3}{Z} + \frac{(Z+C_2C_4)(Z+C_1C_5)}{Z(C_4Z+C_1C_4C_5+C_5Z+C_2C_4C_5)}} \\
&= \frac{1}{\frac{C_3C_4Z+C_1C_3C_4C_5+C_3C_5Z+C_2C_3C_4C_5+(Z+C_2C_4)(Z+C_1C_5)}{Z(C_4Z+C_1C_4C_5+C_5Z+C_2C_4C_5)}} \\
&= \frac{Z(C_4Z + C_1C_4C_5 + C_5Z + C_2C_4C_5)}{Z(C_3C_4 + C_3C_5 + Z + C_1C_5 + C_2C_4) + C_1C_3C_4C_5 + C_2C_3C_4C_5 + C_1C_2C_4C_5} \\
&= \frac{Z(C_4Z + C_5Z + C_1C_4C_5 + C_2C_4C_5)}{Z(C_3C_4 + C_3C_5 + Z + C_1C_5 + C_2C_4) + C_4C_5Z} \\
&= \frac{Z(C_4 + C_5) + C_1C_4C_5 + C_2C_4C_5}{C_3C_4 + C_3C_5 + Z + C_1C_5 + C_2C_4 + C_4C_5} \\
&= \frac{C_1C_2C_4 + C_1C_3C_4 + C_2C_3C_4 + C_1C_2C_5 + C_1C_3C_5 + C_2C_3C_5 + C_1C_4C_5 + C_2C_4C_5}{C_3C_4 + C_3C_5 + C_1C_2 + C_1C_3 + C_2C_3 + C_1C_5 + C_2C_4 + C_4C_5}
\end{aligned}$$

Rearranging the terms in order of subscripts, we state the equivalent capacitance:

$$C_{eq} = \frac{C_1C_2C_4 + C_1C_2C_5 + C_1C_3C_4 + C_1C_3C_5 + C_1C_4C_5 + C_2C_3C_4 + C_2C_3C_5 + C_2C_4C_5}{C_1C_2 + C_1C_3 + C_1C_5 + C_2C_3 + C_2C_4 + C_3C_4 + C_3C_5 + C_4C_5}$$