Example. *A subgroup of a group exists in which more than one but less than all of the left cosets of the subgroup coincide with right cosets of the same subgroup.*

Consider D_4, the Dihedral group of degree 4, that is the group generated by elements a and b such that $|a| = 4$, $|b| = 2$, and $ba = a^{-1}b$.

The Cayley table for D_4 is:

$*$	1	a	a^2	a^3	b	ab	a^2b	a^3b
1	1	a	a^2	a^3	b	ab	a^2b	a^3b
a	a	a^2	a^3	1	ab	a^2b	a^3b	b
a^2	a^2	a^3	1	a	a^2b	a^3b	b	ab
a^3	a^3	1	a	a^2	a^3b	b	ab	a^2b
b	b	a^3b	a^2b	ab	1	a^3	a^2	a
ab	ab	b	a^3b	a^2b	a	1	a^3	a^2
a^2b	a^2b	ab	b	a^3b	a^2	a	1	a^3
a^3b	a^3b	a^2b	ab	b	a^3	a^2	a	1

Let L_X and R_X be the sets of left and right cosets, respectively, of a subgroup X of D_4. We will show that $1 < |L_X \cap R_X| < [D_4 : X]$ when $X \in \big\{\langle b\rangle, \langle ab\rangle, \langle a^2b\rangle, \langle a^3b\rangle\big\}$. To do this, we will show that this inequality holds when $X = \langle a^kb\rangle$ for any integer $k \geq 0$.

First note that $ba^k = a^{-k}b$ for all integer $k \geq 0$, since when $k = 0$ we have the trivial case $b = b$, and if $ba^k = a^{-k}b$ holds for some $k \geq 0$ then $ba^{k+1} = baa^k = a^{-1}ba^k = a^{-1}a^{-k}b = a^{-(k+1)}b$ so that by mathematical induction it holds for all $k \geq 0$.

Now suppose $X = \langle a^kb\rangle$ for some integer $k \geq 0$. Then since $(a^kb)^2 = a^kba^kb = a^ka^{-k}bb = b^2 = 1$, we have $X = \{a^kb, 1\}$.

For left cosets we have,

$$Xa = \{a^kba, a\} = \{a^ka^{-1}b, a\} = \{a^ka^3b, a\} = \{a^{k+3}b, a\}$$
$$Xa^2 = \{a^kba^2, a^2\} = \{a^ka^{-2}b, a\} = \{a^ka^2b, a^2\} = \{a^{k+2}b, a^2\}$$
$$Xa^3 = \{a^kba^3, a^3\} = \{a^ka^{-3}b, a\} = \{a^kab, a^3\} = \{a^{k+1}b, a^3\}$$

so that $L_X = \big\{X, \{a^{k+3}b, a\}, \{a^{k+2}b, a^2\}, \{a^{k+1}b, a^3\}\big\}$.

And for right cosets,

$$aX = \{aa^kb, a\} = \{a^{k+1}b, a\}$$
$$a^2X = \{a^2a^kb, a^2\} = \{a^{k+2}b, a^2\}$$
$$a^3X = \{a^3a^kb, a^3\} = \{a^{k+3}b, a^3\}$$

so that $R_X = \big\{X, \{a^{k+1}b, a\}, \{a^{k+2}b, a^2\}, \{a^{k+3}b, a^3\}\big\}$.

Hence $L_X \cap R_X = \big\{X, \{a^{k+2}b, a^2\}\big\}$ so that $1 < |L_X \cap R_X| < 4 = [D_4 : X]$.