Abstract Algebra: Section 10

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10.16 Problem. *Assume that* $a \equiv b$ *and* $c \equiv d \pmod{n}$. *Prove that* $ac \equiv bd \pmod{n}$.

Suppose that a, b, c, and d, are integers such that $a \equiv b$ and $c \equiv d \pmod{n}$. Then $n \mid a - b$ and $n \mid c - d$. Therefore $a - b = k_1 n$ and $c - d = k_2 n$ for some integers k_1 and k_2. Therefore, multiplying the first by c and the second by b we have

$$\left\{ \begin{array}{c} ac - bc = k_1 nc \\ bc - bd = k_2 nb \end{array} \right\}$$

Then we may add these together to obtain $ac - bd = (k_1 c + k_2 b)n$. Therefore, since k_1, k_2, c, and b are all integers, $n \mid ac - bd$. Therefore $ac \equiv bd \pmod{n}$. **Q.E.D.**

10.20 Problem. *Prove that if* $m \mid n$ *and* $a \equiv b \pmod{n}$, *then* $a \equiv b \pmod{m}$.

Suppose that m, n, a, and b are integers such that $m \mid n$ and $a \equiv b \pmod{n}$. Then $a - b = k_1 n$ and $n = k_2 m$ for some integers k_1 and k_2. Therefore $a - b = k_1 k_2 m$. Therefore, since $k_1 k_2 \in \mathbf{Z}$, $a \equiv b \pmod{m}$. **Q.E.D.**